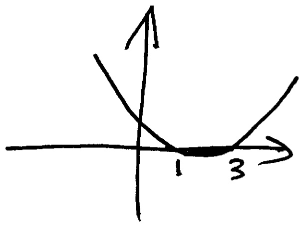
June 2012 FSMQ Add Maths Solu

1. (i) $x^2 - 4x + 3 \leq 0$

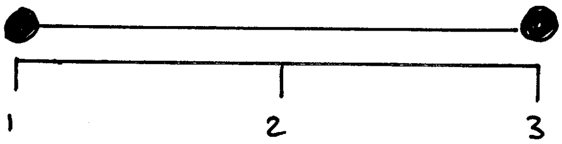
$(x-3)(x-1) \leq 0$

$x = 3 \qquad x = 1$



$1 \leq x \leq 3$

(ii)



2. $P(6) = \frac{1}{5}$

(i) At least 1 six $\rightarrow$ 1 or more

$P(0 \text{ sixes}) = \left(\frac{4}{5}\right)^5$

$\therefore \quad 1 - \left(\frac{4}{5}\right)^5 = \frac{2101}{3125} \quad 0.672 \text{ (3sf)}$

(ii) $P(X=3): \quad \binom{5}{3} \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^2$

$= \frac{32}{625} \quad 0.0512 \text{ (3sf)}$

3. (i) $f(3) = 12$

$3^3 + 3a + 6 = 12$

$3a = 12 - 27 - 6$

$3a = -21$

$\therefore \quad a = -7 \quad$ QED

(ii) $f(x) = x^3 - 7x + 6$

$$f(x) = (x+3)(x-2)(x-1)$$

4. $u = 10\,ms^{-1}$ $\quad v = 16\,ms^{-1}$ $\quad t = 10s$

$$S = \tfrac{1}{2}(u+v)t$$
$$= \tfrac{1}{2}(10+16)\times 10$$
$$= 130m$$

$$V = u + at$$
$$16 = 10 + 10a$$
$$6 = 10a$$
$$\therefore a = 0.6\,ms^{-2}$$

5 (i) $3\cos^2\theta = \sin\theta + 1$

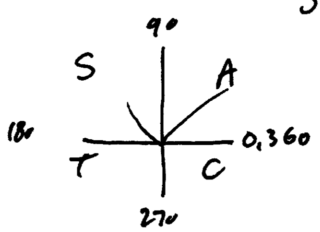
using $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \underline{\cos^2\theta = 1 - \sin^2\theta}$

$$3(1-\sin^2\theta) = \sin\theta + 1$$
$$3 - 3\sin^2\theta = \sin\theta + 1$$
$$0 = 3\sin^2\theta + \sin\theta - 2 \quad \text{QED}$$

(ii) Let $s = \sin\theta$

$$3s^2 + s - 2 = 0$$
$$(3s-2)(s+1) = 0$$

$\sin\theta = 2/3$ or $\sin\theta = -1$



$\theta = 41.8^\circ$ or 138° (3sf) $\qquad \theta = 270^\circ$

6. $y = 2x^3 - 9x^2 + 12x$

(i) $\frac{dy}{dx} = 6x^2 - 18x + 12$

$0 = 6x^2 - 18x + 12$

$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

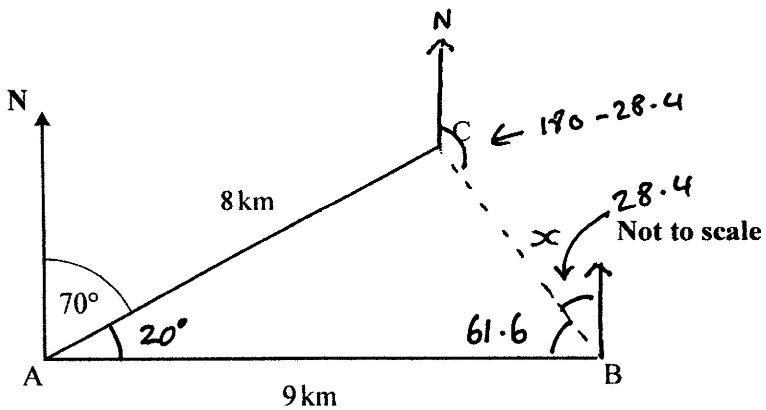
SP at $x = 2$ and $x = 1$

(ii) $\frac{d^2y}{dx^2} = 12x - 18$

At $x = 2$ $\frac{d^2y}{dx^2} = 12(2) - 18$

$= 6$ MIN

7. (i)



$x^2 = 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 20$

$x^2 = 9.68\ldots$

$x = 3.11$ km (3sf)

(ii) $\frac{\sin A\hat{B}C}{8} = \frac{\sin 20}{3.11}$

$\sin A\hat{B}C = \frac{8 \sin 20}{3.11}$ $\qquad A\hat{B}C = 61.6^\circ$ (3sf)

$\therefore$ Bearing of 152°

8. $\int_0^2 (x^2+2x-3)\,dx = \left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x\right]_0^2$

(i) Subs $x=2$ sub $x=0$

$\frac{2^3}{3} + 2^2 - 3(2)$ 0

$\left(\frac{8}{3}+4-6\right) - (0) = 2/3$ QED

(ii) Area below x axis would be negative

(iii) $\int_0^1 (x^2+2x-3)\,dx + \int_1^2 (x^2+2x-3)\,dx$

using part (i)

$\left|\left(\frac{1^3}{3} + 1^2 - 3(1)\right) - (0)\right| + \left(\frac{2^3}{3} + 2^2 - 3(2)\right) - \left(\frac{1^3}{3} + 1^2 - 3(1)\right)$

Absolute value of... $\left|-\frac{5}{3}\right|$ $\frac{2}{3} - \left(-\frac{5}{3}\right)$

$\frac{5}{3} + \frac{2}{3} + \frac{5}{3} = \frac{12}{3} = 4 \text{ units}^2$

9. $h = 7 - 5\cos(480t)^\circ$

(i) $h = 7 - 5\underbrace{\cos(0)}_{1}$

$h = 7-5$

$h = 2\text{m}$

(ii) $\cos(480t)^\circ \rightarrow$ MAX is 1

So $-5\cos(480t)^\circ \rightarrow$ MAX is 5

$-5\cos(480t)^\circ + 7 \rightarrow$ MAX is $\underline{\underline{12}}$

(iii) $9 = 7 - 5\cos(480t)^\circ$

$-\frac{2}{5} = \cos(480t)^\circ$

$\cos^{-1}\left(-\frac{2}{5}\right) = 480t \Rightarrow 113.578... = 480t$

$t = 0.2366...$ mins

$\times 60$

$= 14.19....$

$= 14$ seconds

Section B

10. $A(1, 10)$ $B(8, 9)$ $C(7, 2)$

(i) $M\left(\frac{1+7}{2}, \frac{10+2}{2}\right)$

$M(4, 6)$

(ii) M is centre $|AC| = \sqrt{(7-1)^2 + (2-10)^2} = \sqrt{36+64}$

$= 10$

$(x-4)^2 + (y-6)^2 = 5^2 = 25$

(iii) subs $(8, 9) \Rightarrow (8-4)^2 + (9-6)^2$

$4^2 + 3^2 = 25$

$16 + 9 = 25$ ✓

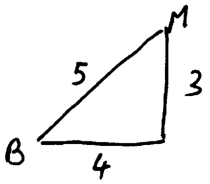
(iv) $A(1,10) \quad M(4,6) \quad B(8,9)$

Grad AM $\quad \dfrac{6-10}{4-1} = -\dfrac{4}{3}$

Grad BM $\quad \dfrac{9-6}{8-4} = \dfrac{3}{4}$

$-\frac{4}{3} \times \frac{3}{4} = -1$ hence perpendicular

(v)



$M(4,6)$

$-4 \quad -3$ $\quad$ Since $|DM| = 5$

$D(0,3)$

11. $y = \frac{1}{2}x^2$

(i) $\dfrac{dy}{dx} = x \qquad$ At $x=-2 \quad \dfrac{dy}{dx} = -2$

so gradient of normal $\dfrac{-1}{-2} = \dfrac{1}{2}$

At $(-2,2)$ $\quad y-2 = \frac{1}{2}(x-(-2))$

$y-2 = \frac{1}{2}(x+2)$

$2y-4 = x+2 \quad \Rightarrow \quad 2y = x+6$

(ii) $\frac{1}{2}x + 3 = \frac{1}{2}x^2$

$3 = \frac{1}{2}x^2 - \frac{1}{2}x$

$0 = \frac{1}{2}x^2 - \frac{1}{2}x - 3$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$\rightarrow x = 3 \quad y = 4.5$

(iii) Top line − Bottom curve

$$\int_{-2}^{3} \left(\tfrac{1}{2}x + 3 - \tfrac{1}{2}x^2\right) dx$$

$$= \left[\frac{x^2}{4} + 3x - \frac{x^3}{6}\right]_{-2}^{3}$$

$$\left(\frac{3^2}{4} + 3(3) - \frac{3^3}{6}\right) - \left(\frac{(-2)^2}{4} + 3(-2) - \frac{(-2)^3}{6}\right)$$

$$= \frac{125}{12} \text{ units}^2$$

12.

(i) $d = av^2 + bv$

$75 = 30^2 a + 30b$

$75 = 900a + 30b$ (1)

$240 = 60^2 a + 60b$

$240 = 3600a + 60b$ (2)

$240 = 3600a + 60b$ −

$150 = 1800a + 60b$

$90 = 1800a$ $a = \frac{1}{20}$

subs to get $b = 1$

$\therefore d = \frac{v^2}{20} + v$ QED

(ii) $d = \dfrac{65^2}{20} + 65 = 276.25$

$d = \dfrac{70^2}{20} + 70 = 315$

$315 - 276.25 = 38.75$ feet

(iii) Let $d = 50$

$50 = \dfrac{v^2}{20} + v$

$1000 = v^2 + 20v$

$0 = v^2 + 20v - 1000$

using quad formula $v = 23.166\ldots$

$v = 23.2$ mph (3sf)

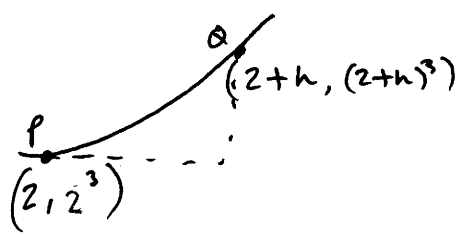
13. (i) $(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots$

$(2+h)^3 = 2^3 + \binom{3}{1} 2^2 h + \binom{3}{2} 2^1 . h^2 + \binom{3}{3} 2^0 h^3$

$= 8 + 12h + 6h^2 + h^3$

$a = 12 \quad b = 6 \quad c = 1$

(ii)



Grad of PQ $\dfrac{(2+h)^3 - 2^3}{2+h-2}$

$$\frac{(2+h)^3-8}{h} \qquad QED$$

(iii) $$\frac{\cancel{8}+12h+6h^2+h^3-\cancel{8}}{h}$$

$$= 12+6h+h^2$$

(iv) $h \to 0$ Grad of $PQ \to 12$

(v) $$(2+h)^4 = 2^4 + \binom{4}{1}2^3h + \binom{4}{2}2^2h^2 + \binom{4}{3}2^1h^3 + \binom{4}{4}2^0h^4$$

$$\frac{\cancel{16}+32h+24h^2+8h^3+h^4-\cancel{16}}{h}$$

$$32+24h+8h^2+h^3$$

As $h \to 0$ Grad of $PQ \to 32$